Ma 17: How to solve it

Mock Putnam 1 — Fall 2026

Session A

Give a complete justification for each answer.

A1. A grasshopper starts at $(0, 0)$ in the plane. Every jump has length 5, and every landing point has integer coordinates. What is the smallest number of jumps with which it can reach $(2026, 2026)$?

A2. A stack contains n cards, labeled $1, 2, \ldots, n$, in an arbitrary order. If the top card has label $k > 1$, reverse the order of the top k cards, leaving all other cards in place. Repeat this operation whenever the top card is not 1. Prove that the process stops after finitely many operations.

A3. Let $f(x)$ be a polynomial with integer coefficients. Set $a_0 = 0$, and

$$a_{r+1} = f(a_r),$$

for every nonnegative integer r. If $a_m = 0$ for some positive integer m, prove that $a_1 = 0$ or $a_2 = 0$.

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Session B

Give a complete justification for each answer.

B1. Let S be a set of positive integers with the following property: if $u, v \in S$, then $u + v \in S$, including when $u = v$. Suppose that exactly $n \geq 1$ positive integers, denoted by $a_1, \ldots, a_n$, do not belong to S. Prove that

$$a_1 + \cdots + a_n \leq n^2.$$

B2. A hotel has 90 rooms and 100 guests. The manager distributes keys before knowing which 90 guests will return that night. Each key opens exactly one room, and multiple copies of a room's key may be issued. The distribution must allow any 90 returning guests to occupy distinct rooms, each using a key that guest received. What is the smallest total number of keys the manager must distribute?

B3. Let $a_1, \ldots, a_n$ be real numbers. Call an index i *good* if there is an index j with $i \leq j \leq n$ such that $a_i + a_{i+1} + \cdots + a_j > 0$. Suppose at least one index is good. Prove that the sum of a_i over all good indices i is positive.