Temporal correlation in LPP with flat initial condition

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Department of Mathematics

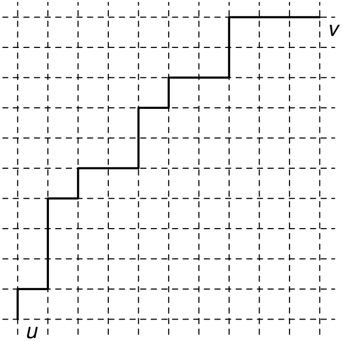
April 29, 2020

Schedule

1. Last passage percolation: background and the problem

2. Proof ideas: geometric arguments and Brownian comparison

Last passage percolation: background and the problem



We study the directed last passage percolation (LPP) on $\mathbb{Z}^2$

- $\omega_v \sim \text{Exp}(1)$, i.i.d. $\forall v \in \mathbb{Z}^2$
- Passage time: $T_{u,v} := \max_\gamma \sum_{w\in\gamma\setminus\{v\}} \omega_w$
- Geodesic: $\Gamma_{u,v} := \text{argmax}_\gamma \sum_{w\in\gamma\setminus\{v\}} \omega_w$

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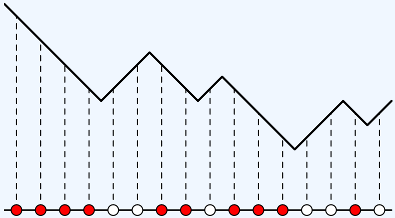
Connections to TASEP



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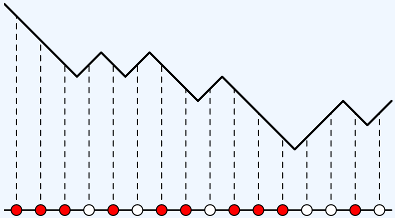

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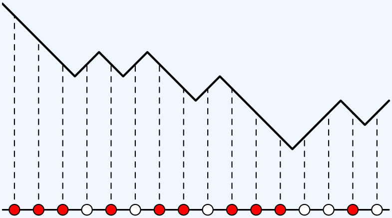
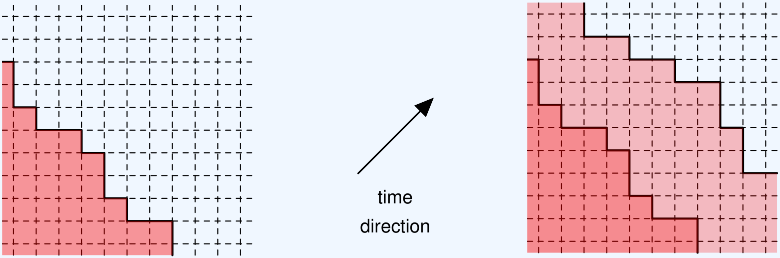


Evolution in exponential LPP:



Time 0: B_0

Time t: $B_t = \{v : \max_{u \in B_0} T_{u,v} < t\}$

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Joint distribution of different end points (space direction):

- Point to line profile (step initial data): stationary Airy_2 process minus a parabola (Borodin and Ferrari, 2008)

$$\mathcal{L}_n(x) := 2^{-4/3}n^{-1/3}\left(T_{(0,0),(n-x(2n)^{2/3},n+x(2n)^{2/3})} - 4n\right) \Rightarrow \mathcal{A}_2(x) - x^2$$

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- Line to line profile (flat initial data): stationary Airy$_1$ process (Borodin, Ferrari, and Prähofer, 2007; Borodin, Ferrari, Prähofer, and Sasamoto, 2007). Let $X_v := \max_x T_{(-x,x),v}$

$$x \mapsto 2^{-4/3}n^{-1/3}\left(X_{(n-x(2n)^{2/3},n+x(2n)^{2/3})} - 4n\right) \Rightarrow 2^{1/3}\mathcal{A}_1(2^{-2/3}x)$$

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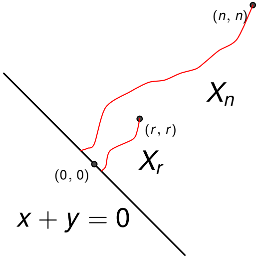
- General initial data: KPZ fixed point (Matetski, Quastel, and Remenik, 2017).

Different time distribution of the passage time



Case of flat initial data

Scale: $r = \tau n$ and $n \to \infty$.

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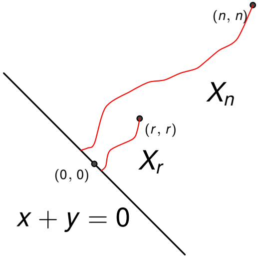

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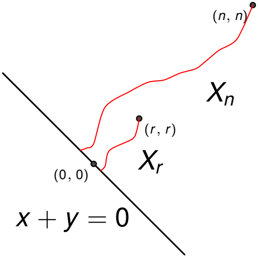

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- Exact asymptotic formulae for the two time distribution: Brownian and geometric LPP (Johansson, 2017, 2019; Johansson and Rahman, 2019), exponential LPP with different initial condition (Baik and Liu, 2019; Liu, 2019).

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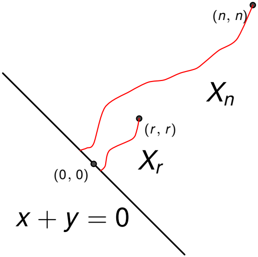

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$$\lim_{n\to\infty} n^{-2/3}\mathrm{Cov}(T_{(0,0),(n,n)}, T_{(0,0),(\tau n,\tau n)}) = \begin{cases} \Theta(\tau^{2/3}) & \tau \to 0 \\ C - \Theta((1-\tau)^{2/3}) & \tau \to 1. \end{cases}$$

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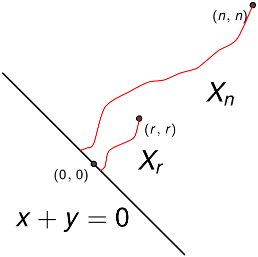

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- The $\tau \to 1$ behaviour is shown to be universal.

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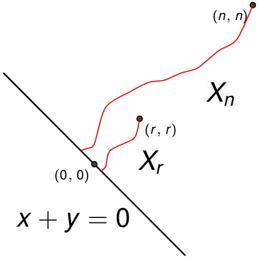

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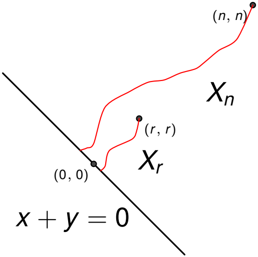

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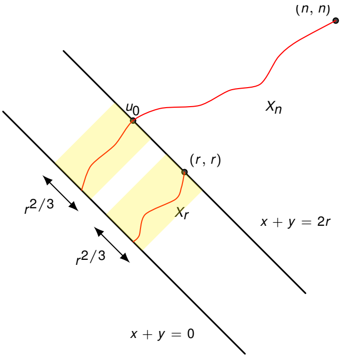
Theorem (Basu, Ganguly, and Z., 2019)

As $\tau \to 0$, we have $\rho(\tau) = \tau^{4/3+o(1)}$.

Proof ideas: geometric arguments and Brownian comparison

Heuristic



Typical: distance between u_0 and (r, r) is $\sim n^{2/3}$; little interaction

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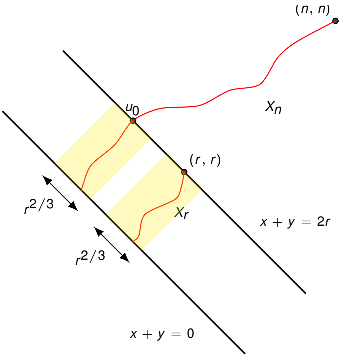

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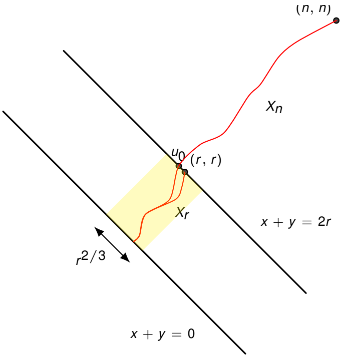

With probability $\sim (r/n)^{2/3}$, overlap for $\sim r$ length, contribute $r^{2/3}$ to covariance

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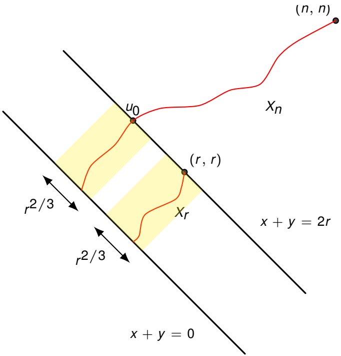

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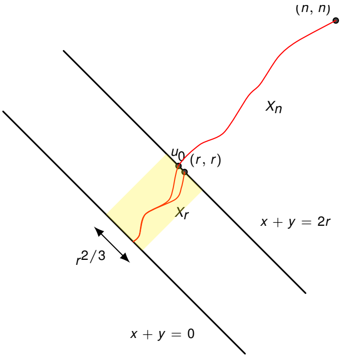

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$$\implies \mathrm{Cov}(X_r, X_n) = \Theta((r/n)^{2/3} r^{2/3})$$

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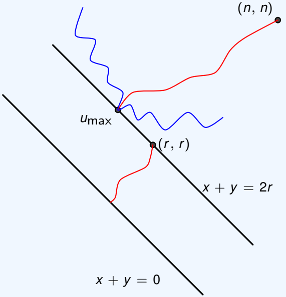

Distance of $u_{\max}$ and $(r, r) \gg r^{2/3}$, and decaying profile: u_0 likely to be far

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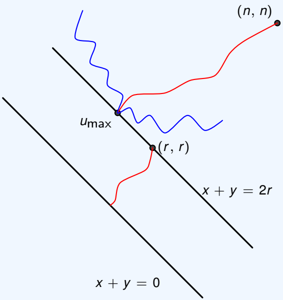

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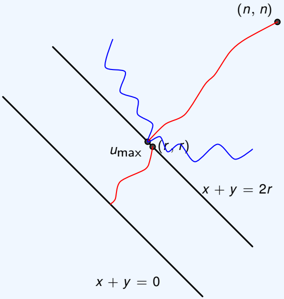

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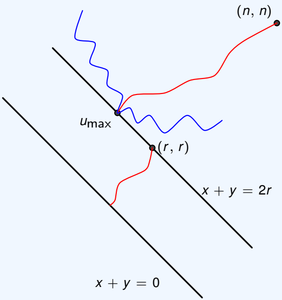

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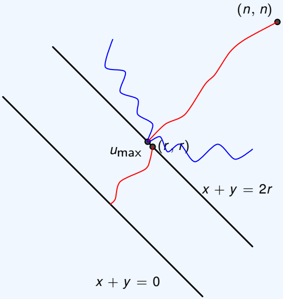

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- Use Brownian comparison of Airy_2 process (Calvert, Hammond, and Hegde, 2019, based on Brownian Gibbs property)

Upper bound

Theorem (Upper Bound)

There exist $C_1, C_2 > 0$ such that for any $\delta \in (0, 1/2)$ there is $n_0(\delta) \in \mathbb{R}_+$ with the following property: for any $n, r \in \mathbb{Z}_+$ with $\delta n < r < \frac{n}{2}$ and $n > n_0(\delta)$ we have

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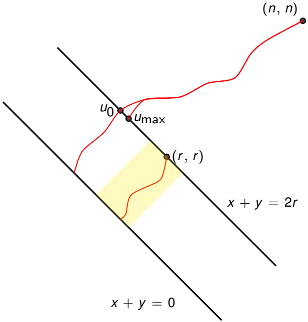

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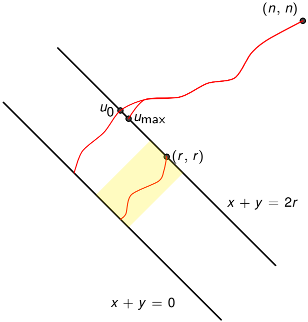

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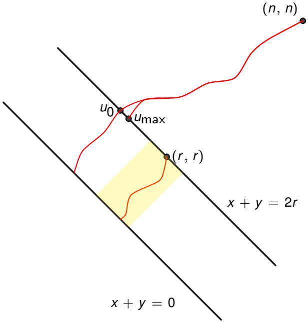

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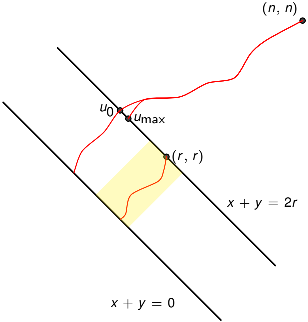

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- As Airy$_2$ process is locally Brownian:
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- Bound Radon-Nikodym derivative of Airy$_2$ over Brownian motion
  $\Longrightarrow$ lose a sub-polynomial factor.

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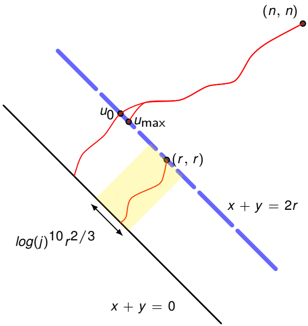

Some technical details:

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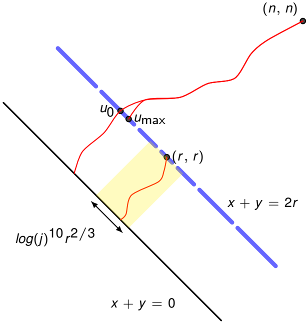

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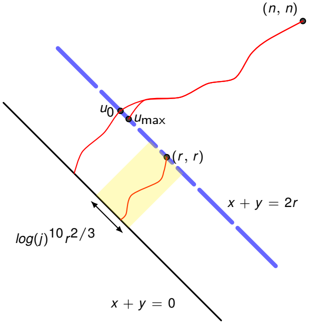

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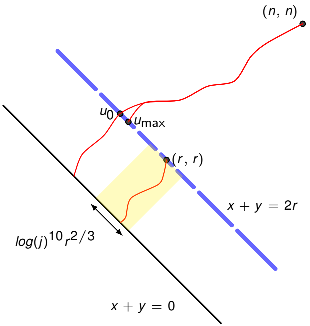

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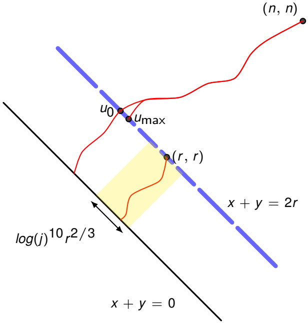

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- Brownian comparison and transversal estimate of geodesics.

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Developed upon arguments for the step initial data case of Basu and Ganguly, 2018.

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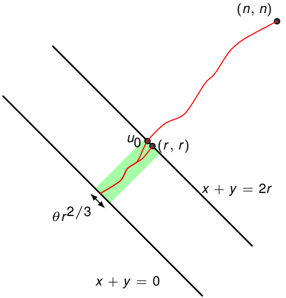
Developed upon arguments for the step initial data case of Basu and Ganguly, 2018.



Theorem (Lower Bound)

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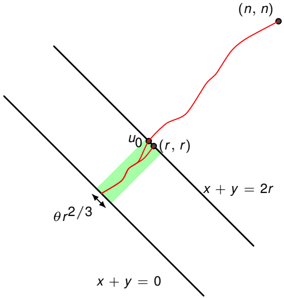

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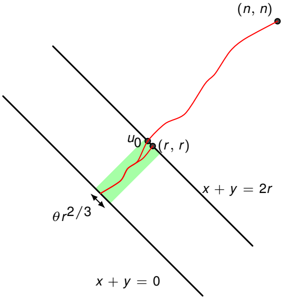
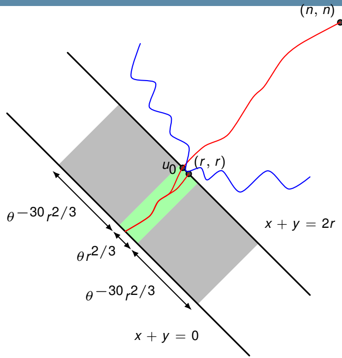


- Idea: construct event with probability $\gtrsim (r/n)^{2/3}$ where coalesce happens.
- Restrict to box of size $\theta r^{2/3} \times r$ $\Longrightarrow$ force coalescing.

Lower bound



- Profile from $x + y = 2r$ to (n, n): $u_{\max}$ within $\theta r^{2/3}$ neighbor of (r, r), and parabolic decay (probability $\gtrsim (r/n)^{2/3}$).

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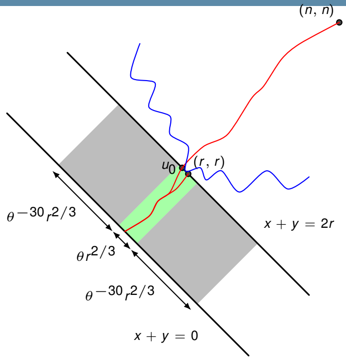

- Profile from $x + y = 2r$ to (n, n): $u_{\max}$ within $\theta r^{2/3}$ neighbor of (r, r), and parabolic decay (probability $\gtrsim (r/n)^{2/3}$).
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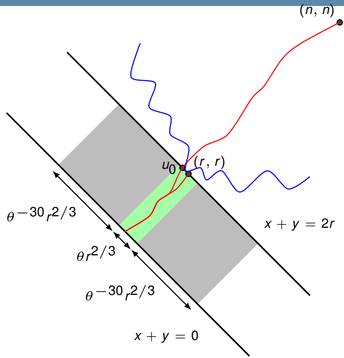

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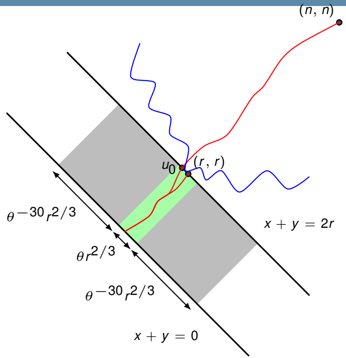

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$\Longrightarrow$ Both X_r and $X_n - X_n^r$ are close to X_θ (best path weight restricted to green box), and $\mathrm{Var}(X_\theta) \gtrsim \theta^{-1/2} r^{2/3}$.

Comments

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- Our arguments are mostly geometric, robust and works for more general initial data.
  i.e. we can replace X_r, X_n by

$$X_r^\pi := \max_x (T_{(-x,x),(r,r)} + \pi(x)) \ , \ \ X_n^\pi := \max_x (T_{(-x,x),(n,n)} + \pi(x)),$$

  where $\pi : \mathbb{Z} \to \mathbb{R}$ satisfies
  (i) $\pi(0) = 0$.
  (ii) $|\pi(x)| \leq C|x|^{1/2-s}$, $\forall x \in \mathbb{Z}$, for some $s \in (0, 1/2)$ and some $C > 0$.

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- For upper bound: possible to remove the factor of $\exp\left(-\log(r/n)^{5/6}\right)$.
  Need better bounds for Brownian comparison of Airy$_2$:

$$P\left[\max_{x\in I} \mathcal{A}_2(x) - x^2 > \max_{x\in[-2M,2M]} \mathcal{A}_2(x) - x^2 - \sqrt{|I|}\right] \leq C|I|,$$

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  Attempt: use formula of Airy_2 process.
  Not easy to analyze, and difficulties in controlling tail events where $\max_{x\in I} \mathcal{A}_2(x) - x^2$ is too large or small.

Thank you!

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